Exciton–phonon coupling and exciton damping due to acoustic phonons in anisotropic nonpolar crystals

Nicolas I Grigorchuk†

Institute for Physics of Semiconductors, 45 Nauka Avenue, Kyiv 252022, Ukraine‡

Abstract. An explicit expression is found for the coupling function of Wannier excitons interacting with both longitudinal and transverse acoustic phonons in anisotropic nonpolar crystals. As an application, the damping of excitons due to the scattering by longitudinal acoustic phonons is calculated.

1. Introduction

The study of the exciton–phonon coupling in anisotropic semiconductors is of great interest, since it constitutes a direct probe of a fundamental interaction in condensed matter physics [1]. In crystals with symmetry lower than cubic, the multiplicity of the degeneracy is lowered and additional possibilities for the exciton motion arise [2]. It seems that a number of effects are directly connected with these possibilities. It was shown by Engelbrecht and Helbig [3] that the crystal anisotropy of 6H-SiC is the cause of the new line in the reflectivity spectrum. An energy splitting of about 2 meV between the $2P_0$ and $2P_{\pm 1}$ exciton states of the A series in AgI due to the anisotropy of this crystal was observed by Dinges *et al* [2]. Numerous other interesting effects had been investigated in earlier publications (see, e.g., [4–8]).

In many applications of exciton–phonon interaction in crystals, the coupling function is used in its simplest form, which is suited only to making qualitative estimates. To study many of the optical characteristics such as the structure of the light absorption bands and its dependence on temperature, and to calculate the exciton lifetime, mean free path etc, it is necessary to know the explicit form of the coupling function [9]. Some details of this problem were reviewed in [10, 11].

A general expression for the exciton–phonon coupling function of the isotropic nonpolar crystal was derived by Ansel'm and Firsov [12] as well as by Toyozawa [13]. This problem for anisotropic systems was considered in more depth by Fock, Kramer and Büttner [14] and by the present author [15] for polar crystals in the case of the excitons interacting with optical phonons.

In the present paper we will obtain an explicit expression for the coupling function of Wannier excitons interacting with acoustic phonons, taking into consideration the anisotropy in the effective mass, the deformation potential constants, the exciton radius and the velocity of sound in the crystal.

† E-mail address: `vakhnenko@gluk.apc.org`.
‡ Mailing address: PB 240, Kyiv-187, UA-252187, Ukraine.

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Section 2 is devoted to the obtaining of the basic formulae for the exciton–phonon coupling in nonpolar anisotropic materials. In section 3 we apply these formulae for the calculation of the exciton damping on acoustic phonons in anisotropic uniaxial crystals.

2. Basic formulae

The interaction of an exciton with vibrations of the anisotropic nonpolar lattice will be treated as an independent interaction of an electron and a hole with longitudinal and transverse acoustic phonons. We shall apply the deformation potential method [16] to write down the energy of the interaction of excitons with acoustic vibrations. The fundamental idea of the deformation potential approach introduced by Shockley and Bardeen [17] is based on the possibility of calculating the scattering of electrons by lattice waves by taking as the interaction potential, $V(r)$, the shift in the energy band $E_1(r)$ resulting from dilation. The justification for this assumption can be easily generalized for anisotropic nonpolar crystals to include the scattering of excitons by longitudinal as well as transverse acoustic modes. When the dilation of the band-edge points is expressed with the help of the field of displacement $U(r)$, the Hamiltonian of the exciton–phonon interaction becomes

$$\hat{H}(r_e, r_h) = -\operatorname{Sp} C_e^{\nu\mu}\epsilon_{\nu\mu}(r_e) + \operatorname{Sp} C_h^{\nu\mu}\epsilon_{\nu\mu}(r_h) \tag{1}$$

where $C_j^{\nu\mu}$ is the tensor of the deformation potential for an electron ($j = e$) or for a hole ($j = h$); $\epsilon_{\nu\mu}$ refers to the tensor function of the mechanical strain. In the above, repeated indices are to be summed over. The symbol Sp means that the summation is carried out over the diagonal elements of the matrix product.

Generally speaking, in an anisotropic medium the velocity of the wave spreading depends on the direction. The formal decompositions of vibrations in both the transverse and longitudinal directions are conditional to some extent, because the polarization vector components $e_\lambda(q)$ (λ refers to the lattice branches, $=1, 2, 3$) in anisotropic media are not exactly directed along or perpendicular to the wave vector q. For an arbitrary q-point one would expect the polarization vector e of the acoustic lattice branches λ to involve longitudinal as well as transverse modes. In anisotropic media the number of elastic constants for the energy density grows with the anisotropy increase, i.e. with the symmetry reduction. This can make the theoretical calculations for an arbitrary anisotropy very complicated. For the sake of simplicity, we will consider only that symmetry for which two of the acoustic branches are still roughly degenerate in one plane, which we suppose is perpendicular to the main optical axis of the uniaxial crystal. That is, we will confine ourselves to the kind of symmetry which in the group theory representation can be described in terms of the one-parameter compact group of two-dimensional rotations $O^+(2)$. This group can be treated as the limit case of the point group C_n with $n \to \infty$. Hence, the symmetry which we wish to consider here will be described by the point group C_∞, and is known as the cylindrical symmetry. For the case in which the cylindrical and the crystal axes coincide, one can always choose those directions in the (x, y) plane for which $\mathbf{C}$ and ϵ from equation (1) may be expressed as

$$\mathbf{C}_j = \begin{pmatrix} C_j^\perp & 0 & 0 \\ 0 & C_j^\perp & 0 \\ 0 & 0 & C_j^\parallel \end{pmatrix} \qquad \epsilon = \begin{pmatrix} \epsilon_\perp & 0 & 0 \\ 0 & \epsilon_\perp & 0 \\ 0 & 0 & \epsilon_\parallel \end{pmatrix}. \tag{2}$$

Instead of the deformation potential components $C^\perp$, $C^\parallel$, the components

$$\Xi_d = C^\perp \qquad \text{and} \qquad \Xi_u = C^\parallel - C^\perp$$

have often been used.

The strain tensor components for the case of a low degree of deformation are given by

$$\epsilon_\perp(\boldsymbol{\rho}_j, z_j) = \frac{\partial \boldsymbol{U}_\perp(\boldsymbol{\rho}_j, z_j)}{\partial \rho} \qquad \epsilon_\parallel(\boldsymbol{\rho}_j, z_j) = \frac{\partial \boldsymbol{U}_\parallel(\boldsymbol{\rho}_j, z_j)}{\partial z} \qquad (3)$$

where $\{\boldsymbol{\rho}_j, z_j\}$ are the cylindrical coordinates of the charge j with the cylindrical axis along z, and $\boldsymbol{U}_\perp$, $\boldsymbol{U}_\parallel$ are the perpendicular and parallel components of the vector field of displacement, which in the quantum representation may be expanded as follows:

$$\boldsymbol{U}(\boldsymbol{\rho}_j, z_j) = \sqrt{\frac{\hbar}{2\rho V}} \sum_{q_\perp, q_\parallel} \frac{1}{(q_\perp^2 + q_\parallel^2)^{1/4}} e^{i(q_\perp \cdot \rho_j + q_\parallel z_j)} \left(\frac{e_{q_\perp}}{\sqrt{S_\perp}} + \frac{e_{q_\parallel}}{\sqrt{S_\parallel}} \right) (\hat{b}_{q_\parallel q_\perp} + \hat{b}^+_{-q_\parallel, -q_\perp}) \qquad (4)$$

where $e_{q_\perp}$ and $e_{q_\parallel}$ are the unit vectors of displacement perpendicular and parallel to the crystal axis directions, $S_\perp$ and $S_\parallel$ refer to the transverse and longitudinal velocities of sound in the crystal, respectively, ρ is the density of the material, V is the volume of the crystal, $b^+_{q_\perp q_\parallel}$ $(b_{q_\perp q_\parallel})$ is the creation (annihilation) operator for a phonon with quasi-momentum components $\{q_\perp,\ q_\parallel = q\}$. As usual, the product of the unit vectors is governed by the Kronecker symbol. Above, we suppose as well that the frequency dependence of an acoustic phonon of the λth branch on the wave vector may be represented as

$$\omega_\lambda(\boldsymbol{q}) = S_\lambda(\boldsymbol{\kappa})|\boldsymbol{q}| \qquad \boldsymbol{\kappa} = \frac{\boldsymbol{q}}{|\boldsymbol{q}|}. \qquad (5)$$

To define the operator of the exciton–acoustic-phonon interaction for a uniaxial crystal, it is convenient to introduce the centre-of-mass ($\boldsymbol{R}$) and relative ($\boldsymbol{\rho}$) coordinates in the system with cylindrical symmetry, as was done in [15]. Then, using equations (3)–(5), the Hamiltonian (1) for the deformation potential can be rewritten in the form

$$\hat{H}_{ac}(\boldsymbol{R}, \boldsymbol{\rho}) = -i\sqrt{\frac{\hbar}{2\rho V}} \sum_{q_\perp q_\parallel} \frac{1}{(q_\perp^2 + q_\parallel^2)^{1/4}} e^{i(q_\perp \cdot \boldsymbol{R}_\perp + q_\parallel \cdot \boldsymbol{R}_\parallel)} \left[e^{iq_\perp \cdot \rho_\perp m_h^\perp / M^\perp + iq_\parallel \cdot \rho_\parallel m_h^\parallel / M^\parallel} w_e(\boldsymbol{q}_\perp, \boldsymbol{q}_\parallel) \right.$$

$$\left. - e^{-iq_\perp \cdot \rho_\perp m_e^\perp / M^\perp - iq_\parallel \cdot \rho_\parallel m_e^\parallel / M^\parallel} w_h(\boldsymbol{q}_\perp, \boldsymbol{q}_\parallel) \right] (\hat{b}_{q_\parallel q_\perp} + \hat{b}^+_{-q_\parallel, -q_\perp}). \qquad (6)$$

Here

$$w_j(\boldsymbol{q}_\perp, \boldsymbol{q}_\parallel) = \frac{C_j^\perp}{\sqrt{S_\perp}}(e_{q_\perp} \cdot \boldsymbol{q}_\perp) + \frac{C_j^\parallel}{\sqrt{S_\parallel}}(e_{q_\parallel} \cdot \boldsymbol{q}_\parallel) \qquad j = \text{e, h}. \qquad (7)$$

m_e^i and m_h^i refer to the ith components of the electron and hole mass, respectively, and $M^i = m_e^i + m_h^i$, $i = \parallel, \perp$. We should expect scattering of excitons to occur due to longitudinal and transverse waves. Therefore, it is necessary to write down the product $e_{q_i} \cdot \boldsymbol{q}_i$ accurately for purely longitudinal ($i = \parallel$) and purely transverse ($i = \perp$) modes. It should be noted here that if the sum of the moduli of the terms in parentheses in equation (7) for the transverse-component mode is negative, then it is necessary to change the signs of the components of $\boldsymbol{q}_\perp$ and $\boldsymbol{q}_\parallel$ to the opposite ones.

Equation (6) is a quite general expression for the deformation potential coupling.

We choose the wave function of the Wannier exciton corresponding to the hydrogen-like model for uniaxial crystal by analogy with the one used in [12, 18], in the form

$$|\boldsymbol{k}_\perp, \boldsymbol{k}_\parallel\rangle = \frac{1}{\sqrt{\pi a_{ex}^2 b_{ex}}} e^{-\sqrt{\rho_\perp^2/a_{ex}^2 + \rho_\parallel^2/b_{ex}^2}} \frac{1}{\sqrt{V}} e^{i(k_\perp \cdot \boldsymbol{R}_\perp + k_\parallel \cdot \boldsymbol{R}_\parallel)} \qquad (8)$$

where a_{ex} and $\boldsymbol{k}_\perp$ are the exciton Bohr radius and the exciton quasi-momentum in the plane (x, y), respectively, and b_{ex} and $\boldsymbol{k}_\parallel$ refer to the same parameters for the z-direction.

Now, using this function, we can average Hamiltonian (6) over the exciton variables. The matrix element calculation in this case is similar to one done for exciton coupling with LO phonons [15]. Taking into account only the linear terms in the operator upon exciton–phonon variables, the operator of the exciton–phonon interaction in the second-quantization representation is

$$\hat{V}_{\text{ex}-(^{\text{L}}_{\text{T}})-\text{ph}} = -\mathrm{i}\frac{(2\pi)^3}{V} \sum_{\substack{q_\perp, q_\parallel \\ k_\perp, k_\parallel}} \Phi_{(^{\text{L}}_{\text{T}})}(q_\perp, q_\parallel)\hat{a}^+_{k_\perp k_\parallel}\hat{a}_{k_\perp+q_\perp, k_\parallel+q_\parallel}(\hat{b}_{q_\perp q_\parallel} + \hat{b}^+_{-q_\perp, -q_\parallel}) \tag{9}$$

where $a^+_{k_\perp k_\parallel}$ ($a_{k_\perp k_\parallel}$) is the creation (annihilation) operator of the exciton with quasi-momentum components $k_\perp$, $k_\parallel$, and

$$\Phi_{\text{L}}(q_\perp, q_\parallel) = \sqrt{\frac{\hbar}{2\rho V}} \frac{1}{(q_\perp^2 + q_\parallel^2)^{3/4}}$$
$$\times \left\{ \frac{q_\perp^2 C_{\text{e}}^\perp/\sqrt{S_\perp} + q_\parallel^2 C_{\text{e}}^\parallel/\sqrt{S_\parallel}}{Z_{\text{e}}^2(q_\perp, q_\parallel)} - \frac{q_\perp^2 C_{\text{h}}^\perp/\sqrt{S_\perp} + q_\parallel^2 C_{\text{h}}^\parallel/\sqrt{S_\parallel}}{Z_{\text{h}}^2(q_\perp, q_\parallel)} \right\} \tag{10}$$

for exciton coupling with longitudinal acoustic vibrations and

$$\Phi_{\text{T}}(q_\perp, q_\parallel) = \sqrt{\frac{\hbar}{2\rho V}} \frac{q_\perp q_\parallel}{(q_\perp^2 + q_\parallel^2)^{3/4}} \left\{ \frac{C_{\text{e}}^\perp/\sqrt{S_\perp} - C_{\text{e}}^\parallel/\sqrt{S_\parallel}}{Z_{\text{e}}^2(q_\perp, q_\parallel)} - \frac{C_{\text{h}}^\perp/\sqrt{S_\perp} - C_{\text{h}}^\parallel/\sqrt{S_\parallel}}{Z_{\text{h}}^2(q_\perp, q_\parallel)} \right\} \tag{11}$$

for exciton coupling with transverse acoustic vibrations. In equations (10), (11) the explicit form of the function Z is as follows:

$$Z_{(^{\text{e}}_{\text{h}})}(q_\perp, q_\parallel) = 1 + \left(\frac{a_{\text{ex}}}{2}q_\perp \frac{m_{(^{\text{e}}_{\text{h}})}^\perp}{M^\perp}\right)^2 + \left(\frac{b_{\text{ex}}}{2}q_\parallel \frac{m_{(^{\text{e}}_{\text{h}})}^\perp}{M^\parallel}\right)^2. \tag{12}$$

The exciton–phonon interaction operator (9) describes the elastic and inelastic scattering of excitons with the emission or absorption of phonons in uniaxial crystals. The number of excitons, like that in isotropic crystals [9], remains unchanged, while the number of phonons changes. The function $\Phi_{\text{L,T}}(q_\perp, q_\parallel)$ represents in an explicit form the exciton–phonon coupling function for excitons interacting with longitudinal (L) and transverse (T) acoustic phonons in uniaxial nonpolar crystals. These functions tends to zero with $q_\perp, q_\parallel \to 0$, simultaneously, and have maxima at small values of both $q_\parallel$ and $q_\perp$. The functions Φ_{L} and Φ_{T} do not depend on the quasi-momenta of excitons or on the exciton bandwidth. In the absence of anisotropy of the crystal ($m_j^\perp = m_j^\parallel$, $C_j^\perp = C_j^\parallel$, $S_\perp = S_\parallel$ and $a_{\text{ex}} = b_{\text{ex}}$), the coupling function (10) coincides with the one found in [9, 12, 13] for isotropic crystals. There may be a discrepancy of a factor of 2/3, because in these papers the deformation potential constant is chosen to be $(2/3)C$ rather than C. For the case of coupling with transverse vibrations, this function, as is seen from (11), is equal to zero.

The formulae for the coupling functions Φ derived above can be applied in quantum field theory, especially in solving the topical optical problems dealing with the exciton–phonon coupling in anisotropic and low-dimensional systems. For instance, the lineshape of light absorption $\mathcal{L}(\omega)$ and the tensor components of the dielectric permeabilities $\varepsilon_{\parallel,\perp}$ of materials may be calculated using these formulae.

3. Exciton damping

As an application of the theory developed above, let us consider the damping in the uniaxial crystal due to exciton scattering by acoustic vibrations.

An external photon entering a crystal can transform directly into an exciton provided that they have the same energy and wavenumber. But this does not constitute absorption, unless this exciton is scattered by a phonon or some other defect in the crystal [19]. The exciton can be scattered by acoustic phonons via the deformation potential or via the piezoelectric potential [20]. Both scattering processes give significant contributions, predominantly for low temperatures. We restrict our consideration to just the first one.

On the other hand, the kind of scattering that we wish to consider here is essentially the scattering of an exciton by emission or absorption of a longitudinal acoustic phonon. The transverse acoustic phonons can be neglected, because shear waves yield a very small deformation potential [13, 21]. The longitudinal acoustic mode plays an important role for intraband scattering [22–25] and gives a contribution comparable to that of the optical mode to the interband scattering [13].

The damping parameter Γ has been introduced by several authors to explain experimental results obtained for the frequency range from that of Raman [22] or Brillouin scattering [23] to that of transmission experiments [24, 25]. We will also calculate this parameter here. The calculations can be performed using the Fermi golden rule. Taking into consideration the coupling function (10) obtained above in the one-phonon approximation for uniaxial crystals, one finds

$$
\Gamma^{\pm}(\omega, T) = \frac{V}{2\pi\hbar} \int_{-\pi/c}^{\pi/c} dq_{\parallel} \int_{0}^{\pi/a} q_{\perp}\, dq_{\perp} \Phi_{\mathrm{LA}}^2(q_{\parallel}, q_{\perp})
$$
$$
\times \left[\bar{n}_{\mathrm{LA}}^{\pm}(q_{\parallel}, q_{\perp}) + \frac{1}{2} \pm \frac{1}{2} \right] \delta[\hbar\omega - E_{q_{\parallel}, q_{\perp}} \mp \Omega_{q_{\parallel}, q_{\perp}}] \tag{13}
$$

where the plus and minus signs correspond to the phonon emission and absorption processes, respectively, c and a are the lattice constants along and transverse to the optical axes of the crystal, respectively,

$$
\bar{n}_{\mathrm{LA}}(q_{\parallel}, q_{\perp}) = \left[\exp\left(\frac{1}{k_B T} \Omega(q_{\parallel}, q_{\perp}) \right) - 1 \right]^{-1}
$$

is the phonon partition function, which obeys Bose statistics, and T is the temperature of the crystal. The dependence of the exciton energies $E_{q_{\parallel}, q_{\perp}}$ on the quasi-momentum will be taken in the form

$$
E_{q_{\parallel}, q_{\perp}} = E_0 + \frac{\hbar^2 q_{\parallel}^2}{2M_{\parallel}} + \frac{\hbar^2 q_{\perp}^2}{2M_{\perp}} \tag{14}
$$

where E_0 is the energy of the exciton band bottom, and we will adopt the following form for the acoustic phonon dispersion:

$$
\Omega_{q_{\parallel}, q_{\perp}} = \hbar \sqrt{S_{\parallel}^2 q_{\parallel}^2 + S_{\perp}^2 q_{\perp}^2}. \tag{15}
$$

Substituting equations (14), (15) into equation (13), we carry out integration over the variable $dq_{\perp}$ first. This is easily done using δ-function. One can see that $\Gamma^{\pm}$ will differ from zero only for

$$
q_{(\mp)}^2 = \left(\frac{M_{\perp} S_{\perp}}{\hbar} \right)^2 \Bigg[2 + \frac{2(\hbar\omega - E_0)}{M_{\perp} S_{\perp}^2} - \frac{M_{\perp}}{M_{\parallel}} \left(\frac{\hbar}{M_{\perp} S_{\perp}} \right)^2 q_{\parallel}^2
$$
$$
\mp 2\sqrt{1 + \frac{2(\hbar\omega - E_0)}{M_{\perp} S_{\perp}^2} + \left(\frac{\hbar}{M_{\perp} S_{\perp}} \right)^2 \left(\frac{S_{\parallel}^2}{S_{\perp}^2} - \frac{M_{\perp}}{M_{\parallel}} \right) q_{\parallel}^2} \Bigg]. \tag{16}
$$

Thus,

$$\Gamma^{\pm}(\omega, T) = \frac{2M_{\perp}V}{\pi\hbar^3} \int_{a_{\mp}}^{b_{\mp}} dq_{\parallel} \left| \frac{d\varphi(q_{\parallel}, q_{\perp})}{dq_{\perp}^2} \right|^{-1}_{q_{\perp}^2 = q_{(\mp)}^2} \Phi_{LA}^2(q_{\parallel}, q_{\perp}) \left[\bar{n}_{LA}^{\pm}(q_{\parallel}, q_{\perp}) + \frac{1}{2} \pm \frac{1}{2} \right]$$
(17)

where $\varphi(q_{\parallel}, q_{\perp})$ is the argument of the δ-function divided by the factor $\hbar^2/2M_{\perp}$. The limits $a_{\mp}$, $b_{\mp}$ may be determined from the condition that $0 \leqslant q_{\mp}^2 \leqslant (\pi/a)^2$. Then, one finds

$$a_{\mp} = 0 \qquad b_{\mp} = \frac{M_{\parallel}S_{\parallel}}{\hbar} \left(\mp 1 + \sqrt{1 + \frac{2}{M_{\parallel}S_{\parallel}^2}(\hbar\omega - E_0)} \right).$$
(18)

The condition of reality of the expressions under the square-root signs in equations (16) and (18) give us, respectively, the new conditions

$$\hbar\omega - E_0 \leqslant -\frac{M_{\perp}S_{\perp}^2}{2} \qquad \hbar\omega - E_0 \leqslant -\frac{M_{\parallel}S_{\parallel}^2}{2}.$$
(19)

These inequalities are valid for the processes with phonon emission, but for the processes with phonon absorption only the condition $\hbar\omega \leqslant E_0$ has to be satisfied.

Performing the integration in equation (17) over $dq_{\parallel}$, one can obtain an explicit expression for $\Gamma^{\pm}(\omega, T)$, which, however, is somewhat cumbersome. We shall here write down just the result that one obtains for small values of the difference $|S_{\parallel}^2/S_{\perp}^2 - M_{\perp}/M_{\parallel}| < 1$.

Let us introduce the anisotropy parameters

$$\alpha = 1 - \frac{M_{\parallel}}{M_{\perp}} \qquad\qquad \beta = 1 - \frac{M_{\parallel}S_{\parallel}^2}{M_{\perp}S_{\perp}^2}$$

$$\gamma_j = 1 - \frac{M_{\parallel}}{M_{\perp}} \frac{C_j^{\parallel}}{C_j^{\perp}} \sqrt{\frac{S_{\perp}}{S_{\parallel}}} \qquad \delta_j = 1 - \frac{M_{\perp}}{M_{\parallel}} \left(\frac{b_{ex}m_j^{\parallel}}{a_{ex}m_j^{\perp}} \right)^2$$
(20)

and the following notation:

$$\alpha_j = \frac{a_{ex}}{2} \frac{m_j^{\perp}}{M_{\perp}} \qquad\qquad \mu = \frac{M_{\parallel}}{M_{\perp}} \frac{S_{\parallel}^2}{S_{\perp}^2}$$

$$\eta_{\parallel,\perp} = \eta(\omega) = 1 + \frac{2}{M_{\parallel,\perp}S_{\parallel,\perp}^2}(\hbar\omega - E_0) \qquad f_{\parallel,\perp}^{\mp} = (\mp 1 + \sqrt{\eta_{\parallel,\perp}(\omega)})^2.$$
(21)

Then,

$$\Gamma^{\pm}(\omega, T) \simeq \frac{M_{\parallel}M_{\perp}^2 S_{\parallel}}{\pi\rho\hbar^4} \sqrt{f_{\parallel}^{\mp}} \frac{\sqrt{f_{\perp}^{\mp}} - (\beta\mu/2\sqrt{\eta_{\perp}})f_{\parallel}^{\mp}}{(\sqrt{\eta_{\perp}} - (\beta\mu/2\sqrt{\eta_{\perp}})f_{\parallel}^{\mp})\left[f_{\perp}^{\mp} - f_{\parallel}^{\mp}\mu(\alpha - \beta/\sqrt{\eta_{\perp}}) \right]^{3/2}}$$

$$\times \left\{ \sum_{j=1}^{2} (-1)^j C_j^{\perp} \frac{f_{\perp}^{\mp} - f_{\parallel}^{\mp}\mu(\gamma_j - \beta/\sqrt{\eta_{\perp}})}{\left[1 + (\alpha_j M_{\perp}S_{\perp}/\hbar)^2(f_{\perp}^{\mp} - f_{\parallel}^{\mp}\mu(\delta_j - \beta/\sqrt{\eta_{\perp}})) \right]^2} \right\}^2$$

$$\times \left\{ \left[\exp\left(\frac{M_{\perp}S_{\perp}^2}{k_BT} \left(\mp 1 + \sqrt{\eta_{\perp}} - f_{\parallel}^{\mp}\frac{\beta\mu}{2\eta_{\perp}} \right) \right) - 1 \right]^{-1} + \frac{1}{2} \pm \frac{1}{2} \right\}$$
(22)

provided that

$$\left| \frac{\beta f_{\parallel}^{\mp}}{\eta_{\perp}} \right| < 1$$
(23)

where $j = 2$ corresponds to the electron mass and $j = 1$ to the hole mass. Equation (22) represents the general expression for the rate of scattering caused by the emission or absorption

of LA phonons in the uniaxial crystal. It allows one to estimate the broadening of the absorption line due to the scattering of excitons on longitudinal acoustic vibrations at any temperatures and for any frequencies (satisfying conditions (19) and (23)), taking account of the anisotropy in the effective masses of the electron and hole, the velocity of sound, the deformation potential constants and the exciton radii for two different directions. It is also important in discussing the dynamical behaviour of excitons in the anisotropic vibration lattice field.

The full damping, which is the sum $\Gamma = \Gamma^+ + \Gamma^-$, is a smooth function which weakly increases with frequency at finite temperatures for the frequency range complying with equations (19), (23), and at zero temperature for the frequencies $\omega \geqslant \hbar^{-1}E_0$. The difference in the starting position for the frequencies follows from the fact that there is an energy shift between the processes with phonon absorption and those with phonon emission. The scattering at the temperature $T = 0$ is only due to the processes with phonon emission, while at temperatures $T \neq 0$ it is caused by phonon absorption as well, and begins with the frequency shifted to the long-wavelength region by a factor of $M_{\parallel,\perp} S^2_{\parallel,\perp}/2$ compared to that for the processes with phonon emission. Such behaviour of Γ is similar to that for the isotropic crystal. To verify this, it is enough to pass to the isotropic case in equation (22), putting $\alpha = \beta = \delta_j = \gamma_j = 0$. Then one gets an equation that exactly coincides with the one obtained earlier for cubic crystals—for instance, in [26].

Equation (22) simplifies for the frequencies $\hbar\omega - E_0 \gg M_{\parallel,\perp} S^2_{\parallel,\perp}/2$. For this case one gets

$$\Gamma^{\pm}_{\mathrm{op}}(\omega, T) \simeq \frac{2M_{\parallel}M_{\perp}}{\pi\rho\hbar^4 S_{\parallel}}(\hbar\omega - E_0)\left\{\sum_{j=1}^{2}(-1)^j C^{\parallel}_j\left[1 + \frac{\hbar\omega - E_0}{2M_{\parallel}}\left(m^{\parallel}_j\frac{b_{\mathrm{ex}}}{\hbar}\right)^2\right]^{-2}\right\}$$

$$\times \left\{\left[\exp\left(-\frac{M_{\perp}S^2_{\perp}}{k_B T}\sqrt{\frac{2(\hbar\omega - E_0)}{M_{\perp}S^2_{\perp}}}\right)\right]^{-1} + \frac{1}{2}\pm\frac{1}{2}\right\}. \tag{24}$$

At low temperatures this expression depends weakly on the temperature, and the damping is mainly due to the Γ^+ processes and depends chiefly on the longitudinal components of the crystal constants M, S, C_j, m_j and b_{ex}.

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